Game against nature: playing on competitive telecommunications services market without knowledge of competitors' costs

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Agenda

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- Basic concepts of Game Theory
- Models of games on telecommunication markets
- Games against nature on telecommunication markets
- Some useful technics of decision support
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Introduction

Introduction to competition on telecomunication markets

- Competition on the telecommunication services market leads to situation, when obtained results by the telecommunications undertakings (operators, service providers - generally market players) depend on their own and other market players' decisions.
- Each player aspires to realize of its own business aims, such as:
  - maximization of profits
  - maximization of market share
  - minimization of costs
  - optimization of trafic distribution
- In general terms we can consider such situations as maximization of payoff function based on **demand** and/or **cost models** with prices of provided services treated as decision variables.

Problem definition

- The aim is to support market players in the setting prices process on detail and wholesale markets (prices for end users and for interconnected undertakings)
- Important difference in setting prices process on detail and wholesale markets:
    - On the detail market players **set** prices
    - On the wholesale market players **negotiate** prices with another players

Basic concepts of Game Theory

Subject of Game Theory

- Game Theory aims at logical analisis of conflict and cooperation situations.
- **Game** is a situation, where:
  - There are at least two players.
  - Each player has a set of admissible **strategies**, that specify the way of his playing.
  - **The result of game** is determined by combinaton of strategies choosen by each of players.
  - Every result of game coresponds to **payoffs** of each player, that can be expressed quantitatively.

Illustration of strategy and payoff concept.

Table: Payoff matrix for two players.

	b_1	b_2	b_3	b_4
a_1			$\vdots$	
a_2	$\ldots\ldots$	$\ldots\ldots$	$[V_3^A(a_2), V_2^B(b_3)]$	$\ldots\ldots$
a_3			$\vdots$	
a_4			$\vdots$	

Models of games on telecommunication markets

Game Theory and competition on the market

- We use Game Theory basic concepts to describe and analyse of decision problems related to seting prices for detail and wholesale services and to construct decision support tools to solve such problems.
- Significant quastions:
  - What is the market game?
  - Who is the player in this game?
  - What is a strategy of the market player?
  - What is a payoff function in this game?

Identitication of the market game

- **Single-criteria market game** - the poin g in two-dimensional space $g \in [M, V]$, where:
  - M - stands for a given telecommunication market (local telephony market, internet servieces market, leased lines market, trafic termination market, etc.)
  - V - stans for criterion of decision evaluation - payoff function (profits, market share, costs, trafic distribution, etc.)

  e.g. g - game for profits on the local telephony market.
- **Player** - telecommunication undertaking as operator or service provider.

Identification of strategy

- **Service Unit** SU_{Aipm} is an elementary part m of services, served by undertaking A in its i^{th} number zone, for the user of profile p, with which is connected the price P_{Aipm}.
- **Strategy** a_j of undertaking A is a set of pairs $\{(SU_{Aipm}, P^j_{Aipm})\}$.

Identification of payoff function

- Four basic payoff functions:
  - Based on demand model:
    - Number of Users - $U = f_U(\mathbf{P_A}, \mathbf{P_B}, \ldots)$
    - Demand for services - $D = f_D(\mathbf{P_A}, \mathbf{P_B}, \ldots)$
  - Based on demand and/or cost models:
    - Costs of services - $K = f_K(D)$
    - Profit - $Z = f_Z(U, D, K)$
- Demand model describes users, so that is common for all telecommunication undertakings. Cost model describes a given undertaking.
- The value of each payoff function depends on prices **P**.

The kinds and features of market games

- A single criterion of evaluation of the game result, concerned on a given market defines a single-criteria market game.
- Each game has the same set of admissible strategies (desicion made in one game influences results in other games).
- Each player **participates** in all games in the sense, that his decisions influence results of all players in all games.
- We say that a given player (A) **plays** in a given game, if he is interested in result of this game.

The kinds and features of market games

- We say that an A player knows **his own** payoff matrix, if he knows his own payoff function, and the set of own and other players' admissible strategies.
- We say that an A player knows other player's payoff matrix, if he knows other player's payoff function and the set of own and other players' admissible strategies.
- To choose decision (strategy) in rational way, an A player should know his own payoff matrix.
- Games in which an A player knows his own and other players' payoff matrix we call in terms of Game Theory **N-person game**.
- Games in which an A player knows only his own payoff matrix we call in terms of Game Theory **games against nature**

The kinds and features of market games

If we assume that players **play** in the same single-criteria games, we cam make the following classification:

- N-person games:
  - games for demand
  - games for number of users
  - games for cost, if the cost models of the other players are known
  - games for profit, if the cost models of the other players are known
- Games against nature
  - games for cost, if the cost models of the other players are unknown
  - games for profit, if the cost models of the other players are unknown

The kinds and features of market games

If players **do not play** in the same game, we can consider the following two situations:

- an A player **knows** in which games the other players play
- an A player doesn't **know** in which games the other players play

The kinds and features of market games

If an *A* player **knows** in which games the other players play, then:

- If an *A* player **knows** the other players' payoff matrixes from the games they are playing in, we can assume that they play in the same game as he, and treat matrixes from games they are playing in as matrixes in this game (it relates to the fact, that all games have the same set of admissible strategies). So that we have **N-person game**.
- If an *A* player **does not know** the other players' payoff matrixes, we treat the game as **game against nature**.

The kinds and features of market games

If an A player **does not know** in which games the other players play, from an A player point of view we tread the situation as a **game against nature**.

The kinds and features of market games

Summary:
An *A* player plays in game against nature, when:

- other players play in games, for which payoff functions are based on a costs model, that are unknown to an *A* player, or
- an *A* player does not know which game other players are playing in.

Games against nature on telecommunication markets

Game against single and double nature

- **Game against double nature** - it is a situation, when:
  - an A player has not set prices for services on detail market, and
  - negotiations of prices for services on wholesale market between an A player and at least one other player are not finished, and
  - at least one other player has not set prices for services on detail market.
- **Game against single nature** - it is a situation, when:
  - Exactly one of above-mentioned conditions are not satisfied.

Game against single and double nature

Conclusion: Setting prices prosess can be considered as multi-stage game, in which the last stage is game against single nature.

Interpretation of a strategy

Table: Payoff matrix of an A player.

	b_1	b_2	b_3	b_4
a_1			$\vdots$	
a_2	$\ldots\ldots$	$\ldots\ldots$	$V_3^A(a_2)$	$\ldots\ldots$
a_3			$\vdots$	
a_4			$\vdots$	

- How to interpret strategies a_i and b_j?

Interpretation of a strategy - game against double nature

In case of game against **double nature**:

- a_i corresponds to prices on an A's detail market
- b_j corresponds both to
  - prices on other players' detail markets, and
  - prices on wholesale markets (possible results of negotiations - strategy of hypothetical players H)

Interpretation of a strategy - game against single nature

In case of game against **single nature**:

1. First case:
   - a_i corresponds to prices on an A's detail market
   - b_j corresponds to
     - prices on a B player's detail market, or
     - prices on wholesale market (possible results of negotiations - strategy of hypothetical player)
2. Second case:
   - a_i corresponds to prices on wholesale market
   - b_j corresponds to prices on a B player's detail market

Some useful technics of decision support

Criteria of chooseing strategy

Commonly known criteria of chooseing strategy:

- Wald criterion

$$\max\{\min_j V_j(a_i) : i \in \mathcal{I}_A\}. \tag{1}$$

- Optimistic criterion

$$\max\{\max_j V_j(a_i) : i \in \mathcal{I}_A\}. \tag{2}$$

- Hurwic criterion

$$\max\{\alpha \cdot \max_j V_j(a_i) + (1-\alpha) \cdot \min_j V_j(a_i) : i \in \mathcal{I}_A\}. \tag{3}$$

Criteria of chooseing strategy

- Laplace criterion

$$\max\{\frac{1}{m}\sum_{j=1}^{m} V_j(a_i) : i \in \mathcal{I}_A\}, \tag{4}$$

- Savage criterion

$$\min\{\max_{j} \widetilde{V}_j(a_i) : i \in \mathcal{I}_A\}, \tag{5}$$

where:

$$\widetilde{V}_j(a_i) = \max_{i} V_j(a_i) - V_j(a_i). \tag{6}$$

Criteria of chooseing strategy

I have proposed some new criteria of chooseing strategy in games against nature, reflecting different models of players' preferences.

Regularisation of ambiguous solusions

Regularisation - reduction of the size of the set of results ambiguous in the sens of one criterion (e.g. Wald), by useing other criterion (e.g. Savage).

Multicriteria analysis

- Regularisation of ambiguous solusions above-mentioned is an exaple of multicriteria optimisation, where:
  - criteria of chooseing strategy (e.g. Wald, Savage, etc.) are treated as criteria of evaluation of solusions
  - scalarized aggregation is made as lexicographic optimisation
- Alternative approach: Reference Point Method
  - criteria of chooseing strategy (e.g. Wald, Savage, etc.) are treated as criteria of evaluation of solusions
  - reference point reflects acceptable and satisying values of such criteria.

The Most Promising Operator Concept

If there is more then 2 competitive players, the question is:

Knowledge of which player's price decision is - in a specific sense - the most valuable?

The Most Promising Operator Concept

General definition:

SQ_A^X - (*Status Quo*) - actually computed outcome for an A player with useing X (e.g. Wald) criterion of chooseing strategy.

KD_O^X - (*Known Decision of player O*) - the outcome computed for an A player with useing X criterion of chooseing strategy, when decision of an O player is known.

VI_{OA}^X - (*Value of Information*) The value of information concerning an O player's decision, computed for an A player with useing X criterion of chooseing strategy.

The Most Promising Operator Concept

- Value of Information

$$VI_{OA}^{X} = |KD_{O}^{X} - SQ_{A}^{X}|. \quad (7)$$

- **The Most Promising Operator** it is such player for which VI_{OA}^{X} has the highest value.

$$Operator\_NO_{A}^{X} = arg \max_{O \neq A} VI_{OA}^{X}$$

Order of players' movemens

In the case of game with two players we have three processes:

- $\mathcal{A}$ - process of setting prices on an A player's detail market
- $\mathcal{B}$ - process of setting prices on a B player's detail market
- $\mathcal{H}$ - process of negotiatons of the pricess on the wholesale market (process of setting prices by a hypothetical player H)

Order of players' movemens

If we place these processes in time, and separate them, then we obtain six variants of playars' movements:

- $\mathcal{ABH}$
- $\mathcal{BAH}$
- $\mathcal{HAB}$
- $\mathcal{HBA}$
- $\mathcal{AHB}$
- $\mathcal{BHA}$

Order of players' movemens

- There are three interesting qustions:
  - Which variant is the best for an A player if he aims at maximization of expected value of payoff function?
  - Which variant is the best if an A player aims at maximization of efficiency of the negotiations process?
  - How much an A player benefits from changeing the order of players' movements?
- We can answer these qustions by analysing an A player's payoff matrix.

The impact of negotiation power and arbitration

- When we analyse the process of negotiation prices on the wholesale market, we should additionaly consider:
  - the impact of negotiation power of the players, and
  - possibility of market regulator arbitration.
- I have started works on these issues.

Directons of the future research

Directions of the future research

Future research are directed on:

- analysis of single-objective **N-person games**
- analysis of multi-objective games

Thank You for attention!

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