

Ineffective Solutions of Two-Person Sequential Non-cooperative Games

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Introduction

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 - Each player has a set of admissible **strategies**, that specify the way of his playing.
 - **The result of a game** is determined by combination of strategies chosen by the players.
 - Every result of a game corresponds to **payoffs** of each player, that can be expressed quantitatively.

Illustration of strategy and payoff concept.

Table: Payoff matrix for two players.

	b_1	b_2	b_3	b_4
a_1			\vdots	
a_2	$[V_3^A(a_2), V_2^B(b_3)]$
a_3			\vdots	
a_4			\vdots	

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- Player A (**first mover**) chooses one of his/her strategy a_i , and next player B (**last mover**) answers with using strategy b_j .
- By ineffective solution we mean that one which **can be improved for one player** without deteriorating the outcome of the other player or which **can be improved for both players**.

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- By ineffective solution we mean that one which **can be improved for one player** without deteriorating the outcome of the other player or which **can be improved for both players**.
- So from at least one player and social point of view it is important to **obtain an effective result** of a game, solution that could not be further improved.

In the following game only one result – $[3, 3]$ is effective.

Table: Game With One Effective Result.

	b_1	b_2
a_1	$[2,2]$	$[1,1]$
a_2	$[3,3]$	$[2,2]$

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- In each game there is **at least one effective solution**.

In some games there could be many effective solutions.

Table: Game With Four Effective Results.

	b_1	b_2
a_1	[1,4]	[2,3]
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	b_1	b_2
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- In this game each result is effective.
- However it is rather an exception: **In most games there are some effective and some ineffective solutions.**

The Reasons of Obtaining Ineffective Results

- 1 Unfavourable structure of the payoff matrix
- 2 Ineffective strategy of the first mover (player A)
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Table: Prisoner Dilemma.

	b_1	b_2
a_1	[3,3]	[1,4]
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- Both players aims at maximizing their own payoffs.

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- Both players have a **dominant strategy**: a_2 and b_2 .
- Dominant strategies lead to an *inefficient result* [2, 2].
- Effective result [3, 3] is difficult to obtain, because both players would like to get **different effective results**: player A – [4, 1] and player B – [1, 4].

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Table: a_2 – Ineffective Move of the Player A . The Cost of Ineffectiveness is Borne by Player B .

	b_1	b_2
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- Player A always obtains the payoff equals to 1.

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- If A chose strategy a_2 , the game would finish with ineffective [1, 2].
- Cost of ineffectiveness is borne only by player B.
- The choice of strategy a_2 can result from an antagonistic motivations of player A or from the fact that player A may not know the payoff matrix of player B.

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Table: a_2 – Ineffective Move of the Player A Resulted from Unknown Payoff Matrix of the Player B.

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a_1	[1,3]	[0,1]
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The cost of ineffectiveness can be also borne by player A.

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- a_2 is the best in the sense of many of the most important criteria of choosing strategy in games against nature.
- a_2 with the answer b_1 leads to ineffective result [1, 2] (better for both would be [2, 3]).

Ineffective move of player A

Antagonistic move of the player A is especially probable if it would lead to high deteriorating of the outcome of player B with small cost for player A like for strategy a_2 .

Table: The Payoff Matrix that Gives an Incentive for Player A for Playing in an Antagonistic Way.

	b_1	b_2
a_1	[10,10]	[8,8]
a_2	[9,1]	[9,0]

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Ineffective solution can be also a result of a fear of player A , that the answer of player B will be antagonistic.

Table: The Payoff Matrix with Strong Incentive for Choosing Strategy a_2 , that Leads to Ineffective Solution if Player A Fears of Antagonistic Answer of Player B .

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- a_2 leads to ineffective result [9, 1].
- a_2 would be chosen if player A was afraid that player B would choose strategy b_2 as the answer for a_1 .

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Ineffective answer of the player B

Ineffective answer ($b_2(a_2) - [0, 2]$) may result from the fact that player B :

- does not know the payoff function of player A ,
- does not want to consider it,
- wants to play in an antagonistic way (antagonistic motives).

Table: The Payoff Matrix with Possible Ineffective Solution if Player B does not Know or does not Want to Consider the Payoffs of Player A .

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Antagonistic motives of player B are simple here to be hidden.

Ineffective answer of the player B

Clear example where an ineffective solution results from antagonistic motives.

Table: Strong Incentive For an Antagonistic Answer – $b_2(a_2)$.

	b_1	b_2
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a_2	[30,30]	[1,29]

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- If player A chose strategy a_2 player B would have strong incentive for playing in an antagonistic way (by choosing strategy b_2).
- If so, the cost of ineffectiveness would be borne by both players, but the cost of player A would be importantly higher.

Results Ineffectiveness Preventing Methods

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Unfavourable structure of the payoff matrix

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- In other words effective solutions are not stable and players have strong incentive for choosing strategy that leads to different solution.

Unfavourable structure of the payoff matrix

Two methods of preventing of the inefficiency in such situations:

- 1 Making a commitment by each side not to choose dominant strategies,

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- ② Eliminating of a dominant strategy from a game.
 - Eliminating of a strategy,
 - Eliminating of a **domination** of this strategy (e.g. changing the form of a payoff function by applying some kind of a penalty function)

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Two cases:

1. Ineffectiveness resulted from the player A 's ignorance of the payoffs of the player B .

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Ineffectiveness Resulted from the Player A 's Ignorance of the Payoffs of the Player B .

- Methods of preventing of the inefficiency in such situations:
 - ① Eliminating strategies that lead to ineffective solutions,
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- Theoretically both methods could be implemented without any intervention by the third side, but for the reason of higher credibility it would be more effective to get help from it.
- There could be a problem with such intervention: **Each method can lead to decreasing the payoff obtained by one of the players.**

Problems with the intervention: better elimination

Table: Passing to Player A Information on the Payoffs of the Player B is Unfavourable for Player B. Favourable for Both Players is Eliminating of the Strategy a_2 .

	b_1	b_2	b_3
a_1	[3,4]	[0,0]	[4,0]
a_2	[1,0]	[2,3]	[4,0]
a_3	[0,0]	[4,2]	[0,0]

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- If strategy a_2 were eliminated then player *A* would choose strategy a_1 , what with the answer b_1 gives the result [3, 4] (effective and better for both players than [2, 3].)
- If *A* got information about payoffs of player *B* then he would choose strategy a_3 . With the answer b_2 it gives the effective but worse for player *B* – [4, 2].

Problems with the intervention: better passing an information

Table: Elimination of the Strategy a_2 Leads to Worse Outcome for Player B . Favourable for B is Passing to A Information on B 's Payoffs.

	b_1	b_2	b_3
a_1	[4,3]	[0,0]	[0,0]
a_2	[3,0]	[1,2]	[4,0]
a_3	[4,0]	[0,0]	[3,1]

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Table: Elimination of the Strategy a_2 Leads to Worse Outcome for Player B . Favourable for B is Passing to A Information on B 's Payoffs.

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a_1	[4,3]	[0,0]	[0,0]
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- The result without intervention: a_2, b_2 – inefficient [1, 2].
- If strategy a_2 were eliminated then player A would choose strategy a_3 , what with the answer b_3 gives the effective result [3, 1] (worse for player B !)
- It would be **better for both players** if player A was informed about B 's payoffs. In such a case player A would choose strategy a_1 , what with the answer b_1 gives the result [4, 3].

Another cases

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- Interference of the regulator can also be not profitable for player A . However it can be like this only in the case of eliminating strategy a_i , leading to inefficiency. Player A never loses on the fact being informed about payoffs of the player B because always he can choose strategy which he wanted without any information.

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- It is also possible that neither of the two methods lead to deteriorating the payoffs of the player B (an example in the article).
- Interference of the regulator can also be not profitable for player A . However it can be like this only in the case of eliminating strategy a_i , leading to inefficiency. Player A never loses on the fact being informed about payoffs of the player B because always he can choose strategy which he wanted without any information.
- It is also probable that **in some cases both methods** – eliminating a given strategy, and passing an information – **can lead to the same, efficient** and better for both players **results** (an example in the article)

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- Sometimes it could be also useful to eliminate one or some of the strategies of player B (after this player A may choose different strategy than before such intervention).
- Sometimes it could be useful **to eliminate simultaneously strategies of player A and B** . More over in some games it could be **necessary** if both methods – eliminating of a strategy (of only one player) and passing an information – leads to the deteriorating of the outcome of one of the players (an example in the article)

Ineffective strategy of the first mover (player A)

Two cases:

1. Ineffectiveness resulted from the player A 's ignorance of the payoffs of the player B .
2. Inefficiency as a result of calculated move of player A (antagonistic motives).

Inefficiency as a Result of Calculated Move of Player A.

Table: The Payoff Matrix that Encourage Player A to Choosing Strategy a_2 that Leads to Inefficient Result.

	b_1	b_2
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- The solution of the problem lays in warming relationships and building higher trust between players.

Inefficiency as a Result of Calculated Move of Player A.

Table: The Payoff Matrix that Encourage Player A to Choosing Strategy a_2 that Leads to Inefficient Result.

	b_1	b_2
a_1	[10,10]	[0,9]
a_2	[9,4]	[9,0]

- The choice of the strategy a_2 can be motivated so by aiming at deterioration of the outcome of the player B, as by „maxmin” approach of the player A, and his fear that player B would play in an antagonistic way.
- The solution of the problem lays in warming relationships and building higher trust between players.
- Eliminating strategy a_2 is not to good approach: generally speaking there is no reasons why a regulator should not trust player A and in the same time trust player B. Better is to eliminate simultaneously a_2 and b_2 .

Results Ineffectiveness Preventing Methods

- 1 Unfavourable structure of the payoff matrix
- 2 Ineffective strategy of the first mover (player *A*)
- 3 Ineffective answer of the last mover (player *B*)

Ineffective strategy of the last mover (player B)

Ineffectiveness of the answer of the player B may result from:

- 1 Ignorance of the payoffs of the player A ;

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Ineffectiveness of the answer of the player B may result from:

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Ineffective strategy of the last mover (player B)

Ineffectiveness of the answer of the player B may result from:

- 1 Ignorance of the payoffs of the player A ;
- 2 Dislike to consider the payoffs of the player A (if they are known);
- 3 Aiming at deteriorating the outcome of the player A .

Ineffective strategy of the last mover (player B)

Table: Possible Inefficient Answer of the Player B For Strategy a_1 .

	b_1	b_2
a_1	[2,2]	[1,2]
a_2	[1,1]	[0,0]

- a_1 dominates strategy a_2

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- For the above mentioned reasons **player B can choose strategy $b_2(a_1)$** , what gives inefficient [1, 2].

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- For the above mentioned reasons player B can choose strategy $b_2(a_1)$, what gives inefficient [1, 2].
- If the problem of ignorance of the payoffs of the player B is the only reason of choosing strategy b_2 , then passing to player B an information about payoffs of the player A would resolve this problem as well as eliminating strategy b_2 .

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- For the above mentioned reasons player B can choose strategy $b_2(a_1)$, what gives inefficient [1, 2].
- If the problem of ignorance of the payoffs of the player B is the only reason of choosing strategy b_2 , then passing to player B an information about payoffs of the player A would resolve this problem as well as eliminating strategy b_2 .
- However, if the answer b_2 comes not from this reason, then the only method of preventing of the inefficiency is to eliminate strategy b_2 .

Ineffective strategy of the last mover (player B)

Table: Difficulty With Interference

	b_1	b_2
a_1	[2,2]	[1,2]
a_2	[2,1]	[1,2]

- Eliminating strategy b_2 would not be a good solution.

Ineffective strategy of the last mover (player B)

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	b_1	b_2
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- Eliminating strategy b_2 would not be a good solution.
- Probably the only methods of preventing of the inefficiency of the solution is to inform the player B about the payoffs of the player A and warming up the relationships between both players, or...

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- Eliminating strategy b_2 would not be a good solution.
- Probably the only methods of preventing of the inefficiency of the solution is to inform the player B about the payoffs of the player A and warming up the relationships between both players, or...
- ...simultaneous elimination strategy of a_2 and b_2 . However such interference in the structure of this game makes that this situation would be no longer a game or even a decision problem situation.

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 - Passing an information to the players on the payoffs of the other side
 - Eliminating the strategies that lead to inefficient result
- Depending on the structure of the payoff matrix these methods lead to different results, sometimes worse for the players than obtained before using them.
- In some cases the solution of the problem of inefficiency of the result lays in warming relationships and building higher trust between players.

Thank you for your attention!

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