Methods of choosing strategy in two person cooperative and competitive market game with unknown aim of the other player

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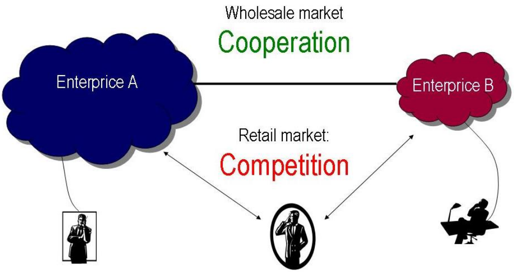
Agenda

1. Problem definition
2. Concepts
3. Methods of choosing strategy

Problem definition

We consider the following problem:

- There are two market players (e.g. telecommunications operators) – A and B.



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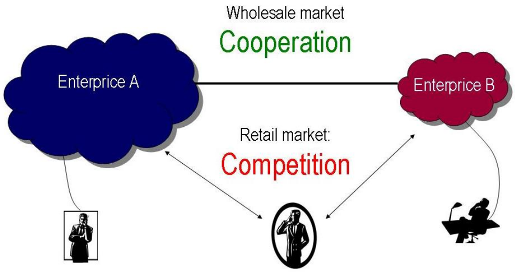

- Players has to compete on the retail market and cooperate on the wholesale market.

Problem definition

- Players don't known the other player aim, which can be neutral, antagonistic or altruistic.
- Each player have to make two important decisions:
  - to set the order of movements of the players,
  - to choose a strategy under a given order of movements.

The general method of dealing with such a problem:

1. Analyse each possible case (each possible order of movements) and indicate, which strategy under such orders should be chosen.
2. The best result of choosing such strategy indicates the best (from a given player point of view) order of movements.

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Problem definition

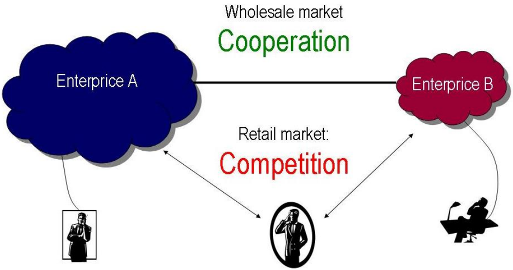
The general problem reduces to:

Indicating the most preferable strategy under each order of movements.

Concepts

Two players - three processes

1. $\mathcal{A}$ – A's retail decision setting process,
2. $\mathcal{B}$ – B's retail decision setting process,
3. $\mathcal{H}$ – negotiation process on the wholesale market between players A and B (the move of a hypothetical player H).



Concepts

Six orders of movements

1. $\mathcal{HAB}$
2. $\mathcal{HBA}$
3. $\mathcal{ABH}$
4. $\mathcal{AHB}$
5. $\mathcal{BAH}$
6. $\mathcal{BHA}$

Double game

Any decisions was made.
Ex. $\mathcal{HAB}$

Single game

One process has been finished.
Ex. $\mathcal{AB}$

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Fig. Payoff matrix for a double game.

h_1

	b_1	b_2	b_3
a_1	[5,5]	[3,4]	[1,3]
a_2	[4,3]	[3,4]	[3,2]
a_3	[3,2]	[3,3]	[2,1]

h_2

	b_1	b_2	b_3
a_1	[4,4]	[4,3]	[2,3]
a_2	[3,2]	[2,3]	[1,1]
a_3	[3,1]	[2,2]	[3,4]

Concepts

- Strategies:
  - a_i - i-th strategy of player A on his retail market
  - b_j - j-th strategy of player B on his retail market
  - h_l - l-th strategy on the wholesale market
- Names of single games:
  - $\mathfrak{a}_i$ - a single game, obtain from a double game by choosing strategy a_i
  - $\mathfrak{b}_j$ - a single game, obtain from a double game by choosing strategy b_j
  - $\mathfrak{h}_l$ - a single game, obtain from a double game by choosing strategy h_l

Concepts

Definition of aims of the players.

- $[V^A_{ijl}, V^B_{ijl}]$ - the result of a game
- Neutral aim of player B

$$\max_{jl} V^B_{ijl}, \forall i \quad (1)$$

- Antagonistic aim of player B

$$\max_{jl}(V^B_{ijl}, -V^A_{ijl}), \forall i \quad (2)$$

- Altruistic aim of player B

$$\max_{jl}(V^B_{ijl}, V^A_{ijl}), \forall i \quad (3)$$

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Case HAB

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Description of case HAB

- Double game (HAB) starts from a negotiation on the wholesale market $\mathcal{H}$
- Single game $\mathfrak{h}_l$ (AB) finishes with the (uncertain) retail decision of player B

A decision problem of player A

- Which strategy h_l during a negotiation process is the best to choose?

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The method:

1. For each single game $\mathfrak{h}_l$ and each strategy a_i determin possible answers b_j of player B, and so possible results $[V^A_{ijl}, V^B_{ijl}]$.
2. Define a skalar measure $\mathcal{V}^A_{ijl}$ of values of each possible result $[V^A_{ijl}, V^B_{ijl}]$ respectively to the aim (neutral, antagonistic or altruistic) of player A.
3. Define the method of agregation (over each b_j) $\Upsilon(\mathcal{V}^A_{il})$ of each skalar values $\mathcal{V}^A_{ijl}$ respectively to the risk aversion of player A.

   Features:

   1. a_i that maximizes $\Upsilon(\mathcal{V}^A_{il})$ for a given single game $\mathfrak{h}_l$ should be choosen during this game.
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An example of using the method.

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Let's assume that player A aims at moderate altruistic goal and evaluates the results $[V^A, V^B]$ accordingly to the equation:

$$\mathcal{V}^A\Big([V^A, V^B]\Big) = w_A \cdot V^A + w_B \cdot V^B, \tag{4}$$

with $w_A = 100$, $w_B = 1$.

Methods of choosing strategy: case HAB

An example of using the method.

h_1

	b_1	b_2	b_3
a_1	505	304	103
a_2	–	304	–
a_3	–	303	201

h_2

	b_1	b_2	b_3
a_1	404	–	203
a_2	–	203	101
a_3	–	202	304

- Scenario h_1-a_1: [505, 304, 103]
- Scenario h_1-a_2: [304]
- Scenario h_1-a_3: [303, 201]
- Scenario h_2-a_1: [404, 203]
- Scenario h_2-a_2: [203, 101]
- Scenario h_2-a_3: [202, 304]

An example of using the method.

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Two general approarches for making an agregation:

1. Assuming full uncertainty about choosing any strategy b_j by player B.

$$\Upsilon(\mathcal{V}_{il}^{A}) = \min_{j} \mathcal{V}_{ijl}^{A}. \quad (5)$$

$$\Upsilon(\mathcal{V}_{il}^{A}) = \frac{1}{J} \sum_{j} \mathcal{V}_{ijl}^{A}. \quad (6)$$

$$\Upsilon(\mathcal{V}_{l}^{A}) = \max_{i} \mathcal{V}_{il}^{A}. \quad (7)$$

2. Assesing probability p_j^{il} of choosing each strategy b_j.

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Assesing probability p_j^{il} bases on two assumtions:

1. If $V^{A*} < V^A$ then $[V^{A*}, V^B]$ is more preferable for player B then $[V^A, V^B]$ - player B would like to play in an antagonistic way
2. The most probable is such strategy b_j that maximizes the measure of incentive for antagonistic move:

$$\tilde{\Upsilon}_{\epsilon il}^{B_j} = \frac{V_{il}^{B_{\max}} \cdot \max\left\{V_{il}^{A_{\max}} - V_{ijl}^{A}, \epsilon\right\}}{V_{il}^{A_{\max}} \cdot \max\left\{V_{il}^{B_{\max}} - V_{ijl}^{B}, \epsilon\right\}}. \quad (8)$$

If so:

$$p_j^{il} = \frac{\tilde{\Upsilon}_{\epsilon il}^{B_j}}{\sum_k \tilde{\Upsilon}_{\epsilon il}^{B_k}}, \quad (9)$$

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$$\tilde{\Upsilon}_{\epsilon il}^{B_j} = \frac{V_{il}^{B_{\max}} \cdot \max\left\{V_{il}^{A_{\max}} - V_{ijl}^{A}, \epsilon\right\}}{V_{il}^{A_{\max}} \cdot \max\left\{V_{il}^{B_{\max}} - V_{ijl}^{B}, \epsilon\right\}}. \tag{8}$$

If so:

$$p_j^{il} = \frac{\tilde{\Upsilon}_{\epsilon il}^{B_j}}{\sum_k \tilde{\Upsilon}_{\epsilon il}^{B_k}}, \tag{9}$$

Methods of choosing strategy: case HAB

- So an aggregator $\Upsilon(\mathcal{V}_{il}^A)$ can be computed as:

$$\Upsilon(\mathcal{V}_{il}^A) = \sum_j p_j^{il} \cdot \mathcal{V}_{il}^A. \tag{10}$$

- For our example we have:
  Scenerio h_1-a_1:

$$\Upsilon(\mathcal{V}_{11}^A) = p_1^{11} \cdot \mathcal{V}_{111}^A + p_2^{11} \cdot \mathcal{V}_{121}^A + p_3^{11} \cdot \mathcal{V}_{131}^A = 263,8.$$

  Scenerio h_1-a_2:

$$\Upsilon(\mathcal{V}_{21}^A) = p_2^{21} \cdot \mathcal{V}_{221}^A = 304.$$

  Scenerio h_2-a_1:

$$\Upsilon(\mathcal{V}_{12}^A) = p_1^{12} \cdot \mathcal{V}_{112}^A + p_3^{12} \cdot \mathcal{V}_{132}^A = 178,25.$$

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Methods of choosing strategy: case HAB

The method:

1. For each single game $\mathfrak{h}_l$ and each strategy a_i determin possible answers b_j of player B, and so possible results $[V^A_{ijl}, V^B_{ijl}]$.
2. Define a skalar measure $\mathcal{V}^A_{ijl}$ of values of each possible result $[V^A_{ijl}, V^B_{ijl}]$ respectively to the aim (neutral, antagonistic or altruistic) of player A.
3. Define the method of agregation (over each b_j) $\Upsilon(\mathcal{V}^A_{il})$ of each skalar values $\mathcal{V}^A_{ijl}$ respectively to the risk aversion of player A.
   Features:
   1. a_i that maximizes $\Upsilon(\mathcal{V}^A_{il})$ for given single game $\mathfrak{h}_l$ should be choosen during this game.
   2. The highest value $\Upsilon(\mathcal{V}^A_{il})$ for a given game $\mathfrak{h}_l$ defines the value of this game.
4. Choose strategy h_l that defines the single game with the highest value.

Methods of choosing strategy: case HAB

Value of single game $\mathfrak{h}_1$

Scenerio h_1-a_1:

$$\Upsilon(\mathcal{V}_{11}^{A}) = 263,8.$$

Scenerio h_1-a_2:

$$\Upsilon(\mathcal{V}_{21}^{A}) = 304.$$

Value of single game $\mathfrak{h}_2$

Scenerio h_2-a_1:

$$\Upsilon(\mathcal{V}_{12}^{A}) = 178,25.$$

Methods of choosing strategy: case HAB

Value of single game $\mathfrak{h}_1$

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Methods of choosing strategy: case HAB

The solution

Because:

$$\Upsilon(\mathcal{V}_{21}^{A}) = 304 > \Upsilon(\mathcal{V}_{12}^{A}) = 178,25.$$

during a negotiation process player A should stress on choosing strategy h_1.

Methods of choosing strategy: case HBA

Case HBA

Methods of choosing strategy: case HBA

Description of case HBA

- Double game (HBA) starts from a negotiation on the wholesale market $\mathcal{H}$
- Single game $\mathfrak{h}_l$ (BA) starts from the (uncertain) retail decision of player B

A decision problem of player A

- Which strategy h_l during a negotiation process is the best to choose?

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Methods of choosing strategy: case HBA

What is new?

Player B doesn't know the aim of the player A and so doesn't know the last move a_i.

Three main approaches:

1. Assuming full uncertainty and treating a double game as a game against nature
2. Assuming that during a single game $\hbar_l$ player B will choose strategy b_j for which the maximal value of coefficient of incentive for antagonistic move of player A - $\Upsilon_{jl}^{A_{\max}}$ will be the smallest.
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Methods of choosing strategy: case HBA

Assuming full uncertainty and treating a double game as a game against nature

The method:

- Define a skalar measure $\mathcal{V}_{ijl}^{A}$ of values of each possible result $[V_{ijl}^{A}, V_{ijl}^{B}]$ respectively to the aim (neutral, antagonistic or altruistic) of player A.
- Transform an oryginal payoff matrix into a matrix with the values of scalar measures $\mathcal{V}_{ijl}^{A}$ instead of the results $[V_{ijl}^{A}, V_{ijl}^{B}]$.
- Transform the matrix with the values of a scalar measure $\mathcal{V}_{ijl}^{A}$ into a game against nature.
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Methods of choosing strategy: case HBA

Fig. Matrix with the values of scalar measures $\mathcal{V}_{ijl}^{A}$.

h_1

	a_1	a_2	a_3
b_1	3	1	4
b_2	2	3	5
b_3	2	4	2

h_2

	a_1	a_2	a_3
b_1	2	3	2
b_2	2	3	4
b_3	3	2	3

h_3

	a_1	a_2	a_3
b_1	5	4	3
b_2	1	5	5
b_3	3	2	3

Methods of choosing strategy: case HBA

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Methods of choosing strategy: case HBA

Fig. Game against nature

	b_1	b_2	b_3
h_1	4	5	4
h_2	3	4	3
h_3	5	5	3

Methods of choosing strategy: case HBA

Assuming that during a single game $\mathfrak{h}_l$ player B will choose strategy b_j for whith the maximal value of coefficient of incentive for antagonistic move of player A - $\Upsilon_{jl}^{A_{\max}}$ will be the smallest.

The method:

- For each game $\mathfrak{h}_l$ and each strategy b_j compute:

$$\Upsilon_{jl}^{A_{\max}} = \frac{V_{jl}^{A_{\max}} \cdot \max\left\{V_{jl}^{B_{\max}} - V_{jl}^{B_{\min}}, \epsilon\right\}}{V_{jl}^{B_{\max}} \cdot \max\left\{V_{jl}^{A_{\max}} - V_{jl}^{A_{\min}} \epsilon\right\}}, \qquad (11)$$

- For each game $\mathfrak{h}_l$ determine:

$$\bar{b}^l = arg\min_j \Upsilon_{jl}^{A_{\max}} \qquad (12)$$

- For each game $\mathfrak{h}_l$ compute a value of a scalar measure $\mathcal{V}_{ijl}^A$ of each possible results $[V_{ijl}^A, V_{ijl}^B]$ puting $b_j = \bar{b}^l$ and a_i respectivelly to real move of player A.
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Methods of choosing strategy: case HBA

Assessing the probabilities p_j^l of choosing strategy b_j.

How to assess probabilities?

- Proportionally to the $V_{jl}^{B_{\min}}$

$$p_j^l = \frac{V_{jl}^{B_{\min}}}{\sum_k V_{kl}^{B_{\min}}}. \tag{13}$$

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$$E(V_l^A) = \sum_j p_j^l \cdot V_{jl}^A(a_i), \tag{16}$$

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- For each game $\mathfrak{h}_l$ compute a value of a scalar measure $E\mathcal{V}_l^A$ of each average results $[E(V_l^A), E(V_l^B)]$ respectivelly to aim (neutral, antagonistic or altruistic) of player A.
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- For each game $\mathfrak{h}_l$ compute a value of a scalar measure $E\mathcal{V}_l^A$ of each average results $[E(V_l^A), E(V_l^B)]$ respectivelly to aim (neutral, antagonistic or altruistic) of player A.
- Choose h_l with the highest value $E\mathcal{V}_l^A$.

Methods of choosing strategy: case HBA

Assessing the probabilities p_j^l of choosing strategy b_j.

The method:

- For each game $\mathfrak{h}_l$ determine probabilities p_j^l.
- For each game $\mathfrak{h}_l$ and each strategy b_j determin the answer a_i
- For each game $\mathfrak{h}_l$ compute the average result $[E(V_l^A), E(V_l^B)]$:

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Methods of choosing strategy: cases AHB and ABH

Cases AHB and ABH

Methods of choosing strategy: cases AHB and ABH

Description of case AHB and ABH

- Double game (AHB and ABH) starts from the retail decision of player A
- Single game $\mathfrak{a}_i$ (HB and BH) consists of two uncertain processes $\mathcal{H}$ and $\mathcal{B}$.

A decision problem of player A

- Which strategy a_l on the retail market is the best to choose?

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In a single game both processes $\mathcal{H}$ and $\mathcal{B}$ are uncertain.

Two main approaches:

1. Assuming full uncertainty and treating a double game as a game against nature
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Thank you for your attention.

Questions?